Error control techniques for wireless communication system

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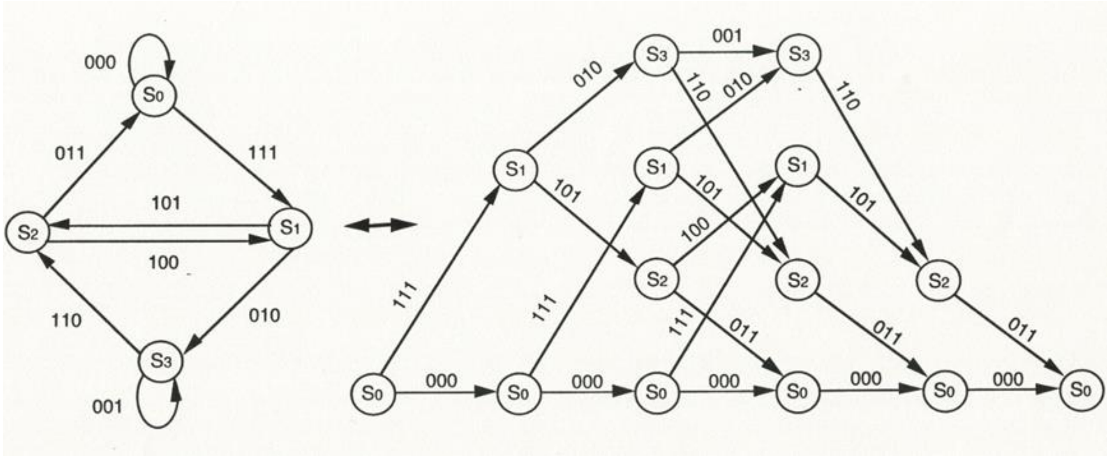
CONVOLUTIONAL CODES

- Convolutional code is a linear forward error-correcting code that generates parity symbols via convolving a data stream with the so-called generating sequences. The dynamic of a convolutional code can be modelled as a finite-state Markov chain which renders a graphic representation called state diagram. If the generator sequences are time-invariant so is the state diagram. A trellis is simply a state diagram that has been extended in time as shown in the figure below.
- Time invariant trellis decoding allows convolutional codes to be maximum-likelihood (ML) soft-decision decoded with reasonable complexity.



The Viterbi Algorithm

- The Viterbi algorithm (VA) is a ML decoding algorithm for linear codes like convolutional codes. VA can be viewed as a trellis decoding algorithm.
- The goal of the VA is to find the path c through the trellis that maximizes P(***r*** | **v**) for a received sequencer **r**. For a length L codeword **v** of a rate $1/n$ code, one has

$$P(\mathbf{r}|\mathbf{v}) = \prod_{i=0}^{L+m-1} P(\mathrm{r}_i|\mathrm{v}_i) = \prod_{i=0}^{N=n(L+m-1)} p(r_i|v_i)$$

$$M(\mathbf{r},\mathbf{v}) = -\log P(\mathbf{r}|\mathbf{v}) = \sum_{i=0}^{N} -\log p(r_i|v_i) = \sum_{i=0}^{N} m(r_i, v_i)$$

where $M(\mathbf{r},\mathbf{v})$ is the distance (path metric) between **r** and **v** while $m(r_i, v_i)$ represents the associated bit metrics.

- The VA finds the candidate codeword (sequence) **v** which is closest to **r**, i.e., the one with the shortest metric.

HIDDEN MARKOV MODEL (HMM)

For example, there are N states S_j and M distinct observations v_k , let A to be State transition probability distribution, B to be Observation symbol probability distribution and π is the Initial state distribution. Then we define Discrete Hidden Markov Model $\lambda = (A, B, \pi)$ to be

$$A = \{a_{ij}\} \quad a_{ij} = P(q_{t+1} = S_j | q_t = S_i) \quad 1 \leq i, j \leq N$$
$$B = \{b_{ik}\} \quad b_{ik} = P(O_t = V_k | q_t = S_i) \quad 1 \leq i \leq N, 1 \leq k \leq M$$
$$\pi = \{\pi_i\} \quad \pi_i = P(q_1 = S_i) \quad 1 \leq i \leq N$$

According to HMM, we can generate an observation sequence $\boldsymbol{O} = \{O_1\ O_2 \ldots O_T\}$ and enumerate every possible state sequence $\mathbf{Q} = q_1 q_2 \ldots q_T$

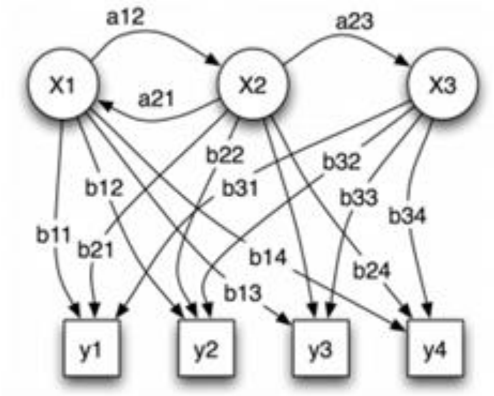


Figure: Discrete HMM with 3 states and 4 possible outputs

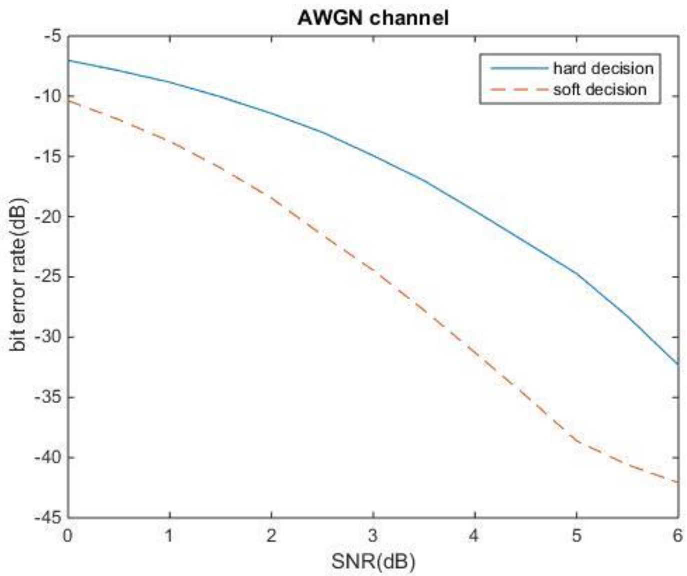
Modelling a wireless channel by a HMM

- We know that the more complex the convolutional code is, the better error-correcting it is, but also more time-consuming. So we choose simple convolutional code for better channel condition, and use complex convolutional code for worse channel condition.

- Transmission over a wireless channel usually has a high dynamic received signal range. The channel variation can also be modelled by a finite state Markov chain in which each state is represented by finite impulse response. Hence a convolutional coded waveform transmitted through a wireless channel can be described by a HMM where each hidden state is a concatenation of a fixed convolutional encoder and a state-specific FIR filter.

Some Preliminary Simulation Results

- The figure on the left-hand side is the BER performance of the simple (2,1,2) convolutional code (see the figure on the right hand side below).
- Hard decision decoding gives BER performance worse than that of the soft decision decoder by about 2 dB, as expected.
- The BER performance is a log-linear function of the reciprocal of SNR when SNR is large.



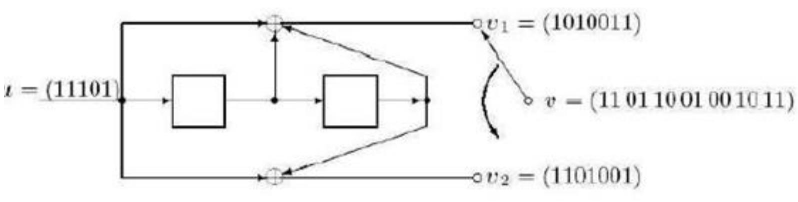


Figure (2,1,2) convolutional code encoder

Figure: bit error rate of convolutional code pass through AWGN channel